Appendix to Why do crises go to waste?

There is a single bureaucracy, which is either a high-productivity type, or a low-productivity type. Given a budget of x, the high type can produce output to a maximum value of $\alpha_H + \beta_H x$, while the low type can produce $\alpha_L + \beta_L x$ with $\beta_L < \beta_H$.[1] This output level can be thought of as the result of an underlying decision problem among the bureau's personnel. For instance, output beyond the default level (α_H or α_L) might be produced by hiring homogenous workers at a unit cost of 1, who produce at β_H or β_L depending on the efficiency of the department's technology. Or, different types of bureaucrats may trade off work and on-the-job leisure differently. However, the bureaucracy may also make a political choice to produce output lower than the maximum. For example, the bureaucracy's manager may allocate money inefficiently so that output is lost. Producing output below the maximum might have a positive cost, since it involves (for example) reorganizing work routines so as to be less efficient, or consuming more leisure than is optimal. I assume for simplicity that this cost is small.[2]

Current spending is at a default level $\bar{x}$ and output is $\bar{y}$, where $\bar{y} = \alpha_H + \beta_H \bar{x} = \alpha_L + \beta_L \bar{x}$ is the point at which both output lines cross. As argued in the text, this is not a coincidence: the government's observation of current productivity can be thought of as reducing its uncertainty over the bureaucracy's output function, perhaps from a much larger set of types.

[1] The linear functional form is not essential: concave output and a weakly convex cost of funds would give similar results. The important condition is that the types' output curves cross only once, at the status quo point.

[2] Relaxing this assumption would have predictable effects. As the cost of producing below the optimum grows, it becomes harder for the low type bureaucracy to pool in the case of budget cuts. Since this cost is difficult to observe empirically, introducing it as a parameter would add little to the model's explanatory value.

The government faces a short-term cost of funds $\hat{c}(x) = kc(x)$. Marginal cost is increasing, since the government's credit in the markets is not unlimited, and since there are competing spending priorities. The parameter k reflects how these conditions may vary. When there are many competing priorities and funds are tight, or when the government must pay high interest rates to borrow, k will be high. I assume $c', c'' > 0$, $c(0) = 0$ and $c'(x) \to 0$ as $x \to 0$. The long-term cost of funds is just $c(x)$.

The timing is as follows:

1. Nature draws the bureaucracy's type $\tau \in \{H, L\}$ which is high (H) with probability π.

2. The government chooses a first-period budget x_1 at a cost $kc(x_1)$.

3. The bureaucracy chooses a level of output $y_1 \leq \alpha_\tau + \beta_\tau x_1$.

4. The government observes y_1 and chooses a second-period budget x_2 at a cost $c(x_2)$.

5. The bureaucracy produces $y_2 = \alpha_\tau + \beta_\tau x_2$.[3]

6. Payoffs are realised. The bureaucracy receives $\delta x_1 + x_2$ and the government's utility is $\delta[y_1 - kc(x_1)] + y_2 - c(x_2)$. Here δ is a parameter reflecting the relative length of the first period, in other words the time until output becomes accurately measurable.

We first look for conditions in which the government can distinguish the types after the first period. If this is so, the second period budget will be x_H solving

[3] Second period productive efficiency is guaranteed by the bureaucracy's small cost of destroying output.

$c'(x_H) = \beta_H$ for a high type and x_L solving $c'(x_L) = \beta_L$ for a low type. Since $x_H > x_L$, each type of bureaucracy would prefer to appear like the high type. For $x_1 \leq \bar{x}$, because the low type's maximum possible output exceeds the high type's, the low type can match any output that the high type could produce. Thus it must be that $x_1 > \bar{x}$. Then the high type bureaucracy can exceed any possible output of the low type, and so identify itself to the government, by producing efficiently.

So for a high enough first period budget, types can always be distinguished. However, if the short-run cost of funds is large, the government may prefer not to do so. The short-run optimum budget allocation, ignoring the second period and assuming that both types of bureaucrat produce efficiently, would be x_1^* solving

$$c'(x_1^*) = \frac{\pi\beta_H + (1-\pi)\beta_L}{k}. \tag{1}$$

If $x_1^* > \bar{x}$ then the short-run optimum will also allow the government to distinguish the types. But if k is large enough this will no longer hold. The government then faces a choice: keep the budget high in the first period in order to observe the bureaucracy's type, or spend less and fail to do so.

Keeping the budget high requires allocating $x_1 = \bar{x}$ in the first period.[4] The government's expected total payoff is then

$$\delta[\bar{y} - kc(\bar{x})] + \pi(y_H - c(x_H)) + (1-\pi)(y_L - c(x_L)) \tag{2}$$

[4] In fact, to learn the department's type we require $x_1 > \bar{x}$. This open set has no minimum, so the government's optimal choice may not be well-defined. This is a purely technical point, and I simply assume that the government must pay some arbitrarily small extra amount above $\bar{x}$ to differentiate the types. An alternative fix would be to add some noise to the department's output. In this case, larger budget increments above $\bar{x}$ would give continuously more accurate signals of deparmental type, and the optimal budget would trade off signal accuracy against period 1 optimality, typically coming in strictly above $\bar{x}$.

where $y_\tau = \alpha_\tau + \beta_\tau x_\tau$ is type τ's output after an optimal choice of period 2 budget.

Alternatively, the government could spend less than $\bar{x}$. If so, then in equilibrium both types will produce the same output. For, if not, the low type could pool with the high type by changing output, and would then increase its period 2 budget. I assume that for $x_1 < \bar{x}$, the high type produces efficiently and the low type matches it. As a result the government's total payoff will be

$$\delta[\alpha_H + \beta_H x_1 - kc(x_1)] + \pi(\alpha_H + \beta_H x_2^*) + (1-\pi)(\alpha_L + \beta_L x_2^*) - c(x_2^*), \quad (3)$$

where x_2^* solves the second period first order condition

$$c'(x_2^*) = \pi\beta_H + (1-\pi)\beta_L. \quad (4)$$

Optimizing over x_1, observe that the government should treat the bureaucracy like a high type, since the low type will pool at the same output. Thus the optimal choice $\hat{x}_1$ solves $c'(\hat{x}_1) = \beta_H / k$.

Comparing the two alternatives, the net benefit of choosing $\bar{x}$ can be split into two components. There is a period 1 negative gain of

$$\delta[\beta_H(\bar{x} - \hat{x}_1) - k(c(\bar{x}) - c(\hat{x}_1))]. \quad (5)$$

This is negative, and decreasing (without bound) in k and δ. (Proof below.) On the other hand there is a period 2 gain from knowing the types, of

$$\pi[\beta_H(x_H - x_2^*) - (c(x_H) - c(x_2^*))] + (1-\pi)[\beta_L(x_L - x_2^*) - (c(x_L) - c(x_2^*))]. \quad (6)$$

This is positive and unrelated to δ or k. Summing these gains, if δ or k is large enough, then the government will strictly prefer to choose $\hat{x}_1$ in period 1, enacting a budget cut which weakens its ability to distinguish an effective from an ineffective department.

Proof that (5) **is negative and decreasing in** δ **and** k**.**

Write (5) as

$$\delta[\beta_H(\bar{x}-\hat{x}_1)-k\int_{\hat{x}_1}^{\bar{x}}c'(x)dx].$$

By the FOC on $\hat{x}_1$, $c'(\hat{x}_1)=\beta_H/k$ and so by $c''>0$, the above is less than

$$\begin{aligned} & \delta[\beta_H(\bar{x}-\hat{x}_1)-k\int_{\hat{x}_1}^{\bar{x}}\frac{\beta_H}{k}dx] \\ = & \delta[\beta_H(\bar{x}-\hat{x}_1)-\beta_H(\bar{x}-\hat{x}_1)] \\ = & 0. \end{aligned}$$

That (5) decreases without bound in δ is then immediate. To show that it decreases in k, suppose $\bar{k}>\underline{k}$. Write $\bar{x}_1$ for the solution to $c'(x)=\beta_H/\bar{k}$, and $\underline{x}_1$ for the solution to $c'(x)=\beta_H/\underline{k}$. Observe that $\bar{x}_1<\underline{x}_1$Then for $\underline{k}$, (5) is

$$\underline{B}\equiv\delta[\beta_H(\bar{x}-\underline{x}_1)-\underline{k}\int_{\underline{x}_1}^{\bar{x}}c'(x)dx],$$

while for $\bar{k}$ it is

$$
\begin{aligned}
& \delta[\beta_H(\bar{x}-\bar{x}_1)-\bar{k}\int_{\bar{x}_1}^{\bar{x}} c'(x)dx] \\
= \quad & \delta[\beta_H(\bar{x}-\underline{x}_1+\underline{x}_1-\bar{x}_1)-\underline{k}\int_{\underline{x}_1}^{\bar{x}} c'(x)dx-\underline{k}\int_{\bar{x}_1}^{\underline{x}_1} c'(x)dx-(\bar{k}-\underline{k})\int_{\bar{x}_1}^{\bar{x}} c'(x)dx] \\
= \quad & \underline{B}+\delta[\beta_H(\underline{x}_1-\bar{x}_1)-\underline{k}\int_{\bar{x}_1}^{\underline{x}_1} c'(x)dx-(\bar{k}-\underline{k})\int_{\bar{x}_1}^{\bar{x}} c'(x)dx].
\end{aligned}
$$

Of the terms in square brackets, the first two sum to less than zero since $c'(\underline{x}_1) = \beta_H/\underline{k}$ and $c'' > 0$, and the last term is negative. Lastly observe that as $k \to \infty$, $\hat{x}_1 \to 0$ and (5) therefore approaches $\delta[\beta_H\bar{x} - kc(\bar{x})]$ which decreases towards infinity with k. QED